Design Steps of Impulse Invariance Method

Step 1 → Analog freq transfer fn $H(s)$ will be given. If it is not given then obtain expression of $H(s)$ from the given specificatio

Step 2 If required expand $H(s)$ by using Partial fraction expansion PFE

Step3 obtain Z transform of each PFE term using impulse invariance transformation equation.

Step4 obtain $H(Z)$ this is required digital IIR filter.

Find out $H(Z)$ using impulse invariance method at 5HZ Sampling freq from $H(s)$ as given below

$$H(s) = \frac{2}{(s+1)(s+2)}$$

Soln Step1 Given analog transfer fn

$$H(s) = \frac{2}{(s+1)(s+2)}$$

Step2 Expand H(s) using Partial fraction expansion as

$$H(S) = \frac{A_1}{(S+1)} + \frac{A_2}{S+2}$$

Poles $P_1 = -1$, $P_2 = -2$

$\boxed{A_1 = 2}$ $\boxed{A_2 = -2}$

$$H(S) = \frac{2}{(S+1)} - \frac{2}{(S+2)}$$

Step3 Now we will obtain Z transform using impulse invariance transformation eq'n

$$\frac{1}{S-P_K} = \frac{1}{1-e^{P_K T_s} \cdot Z^{-1}}$$

T_s = Sampling time. Now given sampling freq is $F_s = 5\,HZ$

$$T_s = \frac{1}{F_s} = \frac{1}{5} = 0.2 \text{ sec}$$

$P_1 = -1$, $P_2 = -2$

$$\frac{1}{S+1} = \frac{1}{1-e^{-1(0.2)} \cdot Z^{-1}} \Rightarrow \frac{1}{1-e^{-0.2} Z^{-1}}$$

$$\frac{1}{S+2} = \frac{1}{1-e^{-2(0.2)}Z^{-1}} = \frac{1}{1-e^{-0.4}\cdot Z^{-1}}$$

Step 4 The transfer fn of digital filter is given by

$$H(Z) = \sum_{k=1}^{N} \frac{A_k}{1-e^{P_k T_s}\cdot Z^{-1}}$$

In this case

$$H(Z) = \frac{A_1}{1-e^{P_1 T_s} Z^{-1}} + \frac{A_2}{1-e^{P_2 T_s} Z^{-1}}$$

using above eq'n's

$$H(Z) = \frac{2}{1-e^{-0.2}Z^{-1}} - \frac{2}{1-e^{-0.4}Z^{-1}}$$

$$H(Z) = \frac{2}{1-0.818Z^{-1}} - \frac{2}{1-0.67Z^{-1}}$$

To convert each term into +ve powers of Z multiplying numerator and denominator of each term by Z we get

$$H(Z) = \frac{2Z}{Z-0.818} - \frac{2Z}{Z-0.67}$$

$$H(Z) = \frac{2Z(Z-0.67) - 2Z(Z-0.818)}{(Z-0.818)(Z-0.67)}$$

$$H(Z) = \frac{2Z^2 - 1.34Z - 2Z^2 + 1.636Z}{Z^2 - 0.67Z - 0.818Z + 0.54}$$

$$H(Z) = \frac{0.29Z}{Z^2 - 1.488Z + 0.54}$$

This is required transfer fn for digital IIR filter.

The system fn of analog filter is given by

$$H_a(S) = \frac{S+0.1}{(S+0.1)^2 + 9}$$

Design IIR digital filter using IIM method

Soln. We have transfer fn $\frac{S+a}{(S+a)^2 + b^2}$

Here $a = 0.1$, $b = 3$.

$$H(Z) = \frac{1 - e^{-0.1 T_s} \cdot Z^{-1} \cos 3T_s}{1 - 2e^{-0.1 T_s} \cdot Z^{-1} \cos 3T_s + e^{-0.2T_s} Z^{-2}}$$ Ans

Convert an analog filter with System fn H(S) into digital IIR filter using IIM.

$$H(s) = \frac{10}{S^2 + 7s + 10}$$

<u>Soln</u>

$$H(S) = \frac{10}{S^2 + 7s + 10}$$

$$H(S) = \frac{10}{(S+5)(S+2)}$$

$$H(S) = \frac{A_1}{(S+5)} + \frac{A_2}{(S+2)}$$

$$A_1 = \frac{10}{-3} \qquad A_2 = \frac{-10}{3}$$

$$H(S) = \frac{-10/3}{(S+5)} - \frac{10/3}{(S+2)}$$

$$\frac{1}{S - P_K} = \frac{1}{1 - e^{P_K T_s} z^{-1}}$$

Let $T_s = 1$

$$\frac{1}{1-e^{-5}Z^{-1}} \Rightarrow \frac{1}{1-6.74\times10^{-3}\cdot Z^{-1}}$$

$$\frac{1}{S+2} = \frac{1}{1-e^{-2}Z^{-1}} = \frac{1}{1-0.135Z^{-1}}$$

$$H(z) = \frac{-10/3}{1-6.74\times10^{-3}Z^{-1}} - \frac{10/3}{1-0.135Z^{-1}}$$